The CSI Journal on
Computer Science and Engineering
Vol. 19, No. 1, 2025
Pages 25-36
doi.org/10.22034/jcse.2025.181898
Survey Paper

Synthetic ECG signal generation, a review

Noushin Riahi[1] Parmida Behain[1]

[1] Computer Engineering Department Alzahra University Tehran, Iran
[2] Noushin Riahi Computer Engineering Department Alzahra University Tehran, Iran

Abstract

An electrocardiogram (ECG/EKG) records the heart's electrical activity and is used to diagnose cardiovascular disease and heart health. Therefore, valuable information about the patient's cardiac condition can be obtained to prevent heart attacks as much as possible. Diagnosing these cardiac diseases requires sufficient data for each category. However, in today's databases, data related to irregular ECG signals are much rarer than regular heartbeats, leading to imbalanced datasets and low accuracy in classifying minority categories. Additionally, patient privacy is also a concern. Considering these two issues, generating artificial ECG signals is essential to balance the dataset and preserve patient privacy. Recently, several techniques have been proposed for generating artificial ECG signals. This article aims to provide an overview of ECG signal generation methods and their related aspects, classify research in this field, and discuss important points and challenges associated with these methods. The evaluation metrics used to assess the generated ECG signals are also explained.
Keywords: ECG signal, Generative adversarial networks (GANs), Auto Encoder, Variational Auto Encoder

1. Introduction

Cardiovascular diseases are one of the most important causes of death in the world. Therefore, it is vital to detect irregular heart rhythms in ECG signals.
Diagnosing heart disease is usually done by medical experts, which is time-consuming and prone to human error. However, automated ECG diagnosis is becoming increasingly acceptable because it not only eliminates random human error but can be available as a bedside test anytime and anywhere.
Nowadays, several datasets have been prepared and made available to the public by collecting and classifying electrocardiograms in hospitals based on the type of disease. Automated ECG detection algorithms can be used to classify ECG beats depending on the overall ECG heart rate pattern. The datasets used to train these classifiers are usually highly imbalanced, as normal heartbeats are more abundant and some abnormal patterns are so rare that the classifier can hardly be trained on them. Therefore, the trained classifier often performs poorly, especially in minority classes with fewer samples [22].
In addition, the automatic diagnosis of these diseases by computers requires a large amount of labeled clinical data for model training and can also preserve the privacy of patients, which is one of the common problems.
On the other hand, Deep Learning has made remarkable advances in the field of healthcare, while the use of deep learning techniques in this domain faces a significant challenge of labeled training data. Acquiring a large amount of labeled data from patients (e.g., EEG, ECG) for labeling by a medical expert is time-consuming and costly. Moreover, physiological data is almost always imbalanced, such as EEG signals of epileptic seizures or ECG signals of heart attacks. Short-term data from abnormal events (such as heart attacks or seizures) are not sufficient to train a conventional supervised classifier [25].
Therefore, there is a need to generate artificial training data to reduce imbalances in ECG datasets and improve the performance of machine learning and deep learning classifiers in patient monitoring systems [25].

There are various methods that can be used to balance imbalanced datasets such as oversampling and undersampling, sampling with artificial data generation, cost-sensitive approaches, active learning techniques, etc. However, these methods come with certain drawbacks. For instance, undersampling runs the risk of discarding potentially valuable data, while oversampling can lead to overfitting as it may create exact duplicates of existing samples. Additionally, the generated data resulting from oversampling techniques are derived from local information and fail to capture the overall distribution of the minority class. Therefore, the original data distribution may not be preserved [25].

However, newer techniques are now being used that can preserve the real distribution of the data. In general, there are several approaches to deal with the imbalance in ECG signal datasets. Among these approaches, Generative Adversarial Networks (GAN), Variational/Autoencoder (V/AE) models, wavelet transform, and others can be utilized for generating synthetic data. After generating artificial ECG signals and evaluating the generated signals, they can be used to address the mentioned problems and achieve better results and high efficiency in the diagnosis of heart diseases.

The paper is organized as follows. In Section II, the ECG signal and its features are described. In Section III, the applications of generating these signals are presented, along with an explanation of the importance of ECG signal generation. Furthermore, Section IV discusses the main challenges in ECG signal generation. Two primary approaches based on existing research and proposed methods are mentioned in Section V for ECG signal generation and they are also summarized in a table format. In Section VI, commonly used datasets for generating ECG signals, along with their features, are provided. Finally, Section VII presents existing evaluation metrics for assessing the accuracy of generated ECG signals and their similarity to real ECG signals.

2. ECG Waveform

The ECG signal records the electrical activity of the heart, which is obtained through electrodes placed on the human body. The signals are received by metal electrodes attached to the limbs and chest and are amplified and recorded by an electrocardiogram (ECG) machine. Therefore, 12-lead ECG and 1 long lead are obtained from the individual. The first 6 leads are responsible for recording the signals from the hands and the feet (limb leads), and the second 6 leads are related to the frontal wires of the heart (chest leads). In addition, a longer lead is usually taken from Lead II to assess the health of the heart rhythm over time. A cardiac cycle denotes the complete sequence of the heart's activity starting from one heartbeat to the onset of the next. It encompasses distinct waves, namely P, T, and QRS complex waves (Q, R, S peaks). The size, configuration, and position of each of these waves hold significant diagnostic value, providing essential information about the heart's performance and indicating the presence or absence of specific diseases [2]. Figure 1 illustrates a cardiac cycle with the mentioned waves.

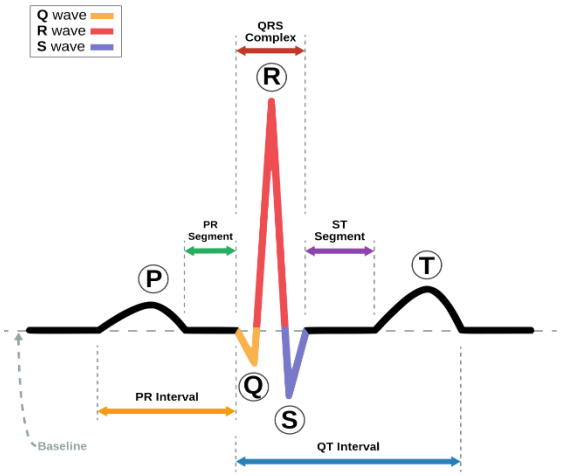

Fig. 1. A cardiac cycle

Electrocardiography (ECG) is used to diagnose cardiac abnormalities, functional disorders, and cardiac arrhythmia. A heartbeat is also known as the cardiac cycle, which is the period that begins with the contraction of the atria and ends with the relaxation of the ventricles [24].

3. Why ECG signal generation?

An ECG signal contains many useful information which can be used for the computer-aided diagnosis of heart diseases, which helps reduce human errors in diagnosing cardiac conditions and for determining individuals' emotions while watching a film using ECG signals. These emotions can include fear, excitement, happiness, anxiety, etc.

According to this, the importance of generating these signals is as follows:

1) Solving the problem of medical data imbalance:
Biophysiological data (such as ECG and EEG) are almost always imbalanced, and generating synthetic signals is done to enhance the performance of deep learning models [25]. In ECG data, the data related to cardiac abnormalities are rare compared to normal heartbeat data, which creates an imbalance and reduces the classification performance in ECG signals [12].

2) Solving the problem of people's privacy and the sensitivity of medical data:
Due to the sensitive nature of medical data, strict control, and regulation govern its access and release. For secondary purposes like software testing or clinical training, it is essential to anonymize the required medical data. Common de-identification methods include generalization, randomization, or pseudonymization. However, it has been demonstrated that merely de-identifying medical data does not guarantee complete privacy protection for individuals, as re-identification is possible through data linkage or residual information. An alternative privacy-preserving approach involves generating synthetic data. The challenge lies in creating data that accurately represents real medical data while safeguarding the privacy of individuals in the original dataset.

If successfully achieved, such synthetic data can be shared and published without privacy concerns, facilitating further research and clinical training purposes. This method offers a promising avenue for striking a balance between data utility and individual privacy protection in the medical domain [7], [18].

3) Solving the problem of needing a lot of data in deep learning methods:

One of the primary challenges faced by these methods is the scarcity of labeled training data. Obtaining a large volume of ECG and EEG data that is expertly labeled is a time-consuming and expensive process [25]. The effectiveness of algorithms can significantly vary across different clinical conditions, highlighting the necessity for a substantial amount of annotated data to ensure their reliability. In other words, without ample support from labeled data, even if algorithms perform well on the available dataset, their real-world applicability may be limited. Consequently, generating realistic ECG signals becomes highly valuable in aiding researchers to enhance their algorithms' performance in ECG signal processing. By generating artificial and realistic samples, researchers can achieve multiple objectives. Firstly, they can augment the existing data, obtaining a rich set of samples that were not initially present in the original database, thus enriching the training data. Secondly, they can acquire samples with specific characteristics, allowing for targeted analysis and investigation. This approach empowers researchers to improve their algorithms' robustness and accuracy, ultimately benefiting the field of ECG signal processing [6].

4. Challenges

The challenges in generating ECG signals are as follows:

1) Generating standard 12-lead ECG signals that are physiologically acceptable [5], [20]. As previously mentioned, standard ECGs consist of 12 leads, with 6 leads corresponding to limb leads (arms and legs) and 6 leads related to the chest. However, most studies have only generated one or two leads based on the available dataset, making the generation of these standard 12-lead ECGs a challenge.

2) Assessing whether the generated ECG signals resemble real ECG signals or not [5]. To validate the accuracy of generated ECG signals, a comparison with real signals is necessary to determine if the generated signals possess the characteristics of real signals.

3) Establishing independent and automated evaluation metrics for the generated signals [5]. Currently, there is no specific evaluation metric available to assess the generated signals, and there is a need to define and identify appropriate evaluation criteria to verify the similarity of the generated signals to real signals.

4) Creating diversity among the generated ECG signals and distinguishing them from each other [5]. Diversity should be incorporated into the generated signals to cover various samples of different diseases or classes, considering the physical conditions of patients. Therefore, the generated signals should encompass different types to create a larger dataset for signal classification.

5) The variable and dynamic nature of ECG signals and their morphological characteristics differ among individuals and are highly dependent on the patient's physiological state. Even for healthy individuals, the R-R interval varies over time within one heartbeat [15], [24]. This implies that most individuals generate a cardiac cycle and repeat it to sustain the heartbeat, while the R-peak intervals during the cardiac cycle differ for each person.

6) Generating ECG signals related to cardiac diseases, where most generation methods either omit disease-related information or create separate models for generating different categories of ECG signals [20]. Studies often use different models separately to generate various types of ECG signals, whereas creating a unified model to generate different types of heartbeats is essential but challenging.

5. ECG generation methods

There are two main approaches for generating ECG signals that are shown in Figure 2:

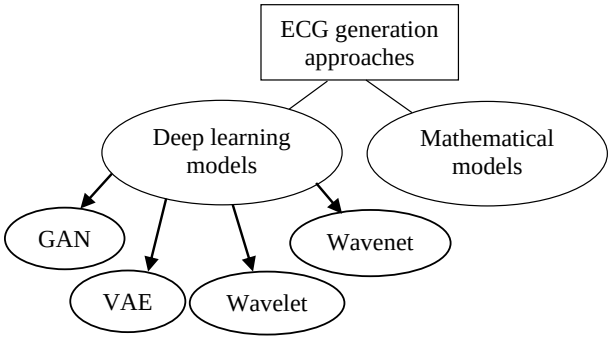

Fig. 2. ECG generation methods

1) Mathematical Models: A mathematical model for ECG (Electrocardiogram) generation refers to a set of mathematical equations and algorithms designed to simulate and produce synthetic ECG signals. The purpose of such models is to mimic the electrical activity of the heart and generate ECG waveforms that resemble real ECG recordings. For a mathematical model-based approach, McSharry. [38] proposed a dynamic model based on three ordinary differential equations that can generate synthetic electrocardiogram signals. M. A. Quiroz-Juárez.[39] proposed a model based on a discretized reaction-diffusion system for producing a set of three nonlinear oscillators which can be used to model the electrical activity of the heart. Jielin Qiu. [40] also presented a new method for augmenting data to balance imbalanced ECG datasets called Optimal Transport(OT). It is a branch of mathematics that focuses on the geometric properties of probability spaces and also it uses the Wasserstein metric between probability distributions. They use this to push samples from the distribution of a majority class to a minority class

so normal heartbeats were considered as the majority category for transport to each minority category.

2) *Deep Learning Models: Generating models based on deep learning are widely used recently and has good performance and accuracy in generating ECG signals. Among the models based on deep learning, different types of GAN, Autoencoder, Wavnet, and Transformer-based Models can be mentioned.*

5.1 . Generative Adversarial Network (GAN) Approach

A Generative Adversarial Network (GAN) is a specific type of neural network architecture. GANs have been remarkably successful in various applications, such as image synthesis, image-to-image translation, super-resolution, style transfer, and more. They have also sparked interest in the creation of deepfake technology, which can be both beneficial and concerning depending on its usage. They have opened up exciting possibilities in the field of artificial intelligence and have been widely adopted in the research community and various industries.

The methods commonly used for generating ECG signals are predominantly based on GAN (Generative Adversarial Networks), which consist of two main components: a generator network (G) and a discriminator network (D). The generator network G is a neural network that accepts random noise z ∈ R as input and produces synthetic data. Discriminator D is a neural network that assesses whether the generated data is real or fake. A general overview of a simple GAN is shown in Figure 3.

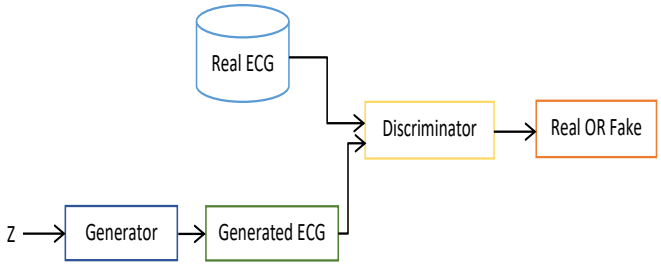

fig. 3. A simple GAN for generating ECG signal

The objective of the generator is to maximize the discriminator's failure to differentiate, while the objective of the discriminator is to minimize its failure. G and D are trained simultaneously in a Minimax game and compete with each other. When the GAN reaches an equilibrium, it converges to an optimal point for the minimax equation. At this stage, the GAN has learned to generate data that resembles real data from the training set. The value function V (G, D) is defined as follows.

$$\min_G \max_D V(G,D) = E_{x\sim p_{dt}(x)}[log\, D(x)] + E_{z\sim p_z(z)}\left[log\left(1 - D(G(z))\right)\right]$$

where x represents real data, z represents the input noise to the G network, G(z) represents the data generated by G, and D(x) represents the probability that the discriminator correctly evaluates the data. D(G(z)) represents the probability that the discriminator correctly identifies the generated data by generator Gas real data [7], [11].

This function is a crucial component that guides the training process and enables the generator and discriminator to learn their respective tasks effectively.

The proposed GAN models in this application can be divided into two groups: conditional and unconditional models.

Conditional GAN models are trying to use auxiliary information as input. It means that in addition to the noise that is given as input to the generator, auxiliary information is also given to the generator network so that based on that auxiliary information such as the class label, it produces the ECG signal associated with that class label. Also, this auxiliary information is given as input to the discriminator so that in addition to predicting whether the input signal is real or not, it can also predict the class label of the ECG signal.

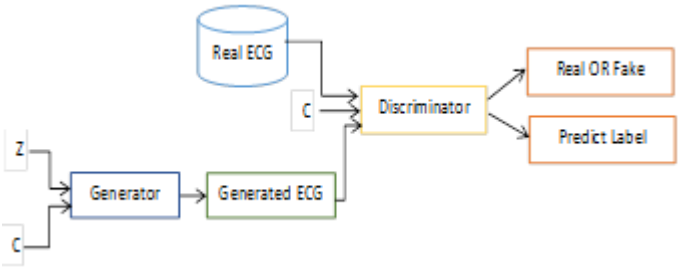

Fig. 4. A Conditional GAN

While the unconditional GAN network produces ECG signals without auxiliary information and only with input noise. It also should be trained separately to produce a signal in each category of the dataset.

GAN-based methods are divided into the following two categories:

1. GAN with signals in the time domain:

In recent years various architectures have been proposed for GANs, some of which have been used for generating ECG signals. These include a combined framework of two GAN networks and also as a single GAN network.

Hybrid models that include two GAN networks use one network to generate the heart rate variability (HRV) pattern and the other one to generate the ECG signal morphology. ECG signal morphology includes P, R, Q, S, and T peaks, and heart rate variability (HRV) is the change in the interval between beats, which is determined as the inverse of the R-R peak interval. Then by combining and merging these two networks, they produce the ECG signal. Rohan Banerjee. [14] proposed a model based on a GAN network that produces ECG signals of the type of Atrial fibrillation (AF) disease, which is a type of cardiac arrhythmia. Their GAN architecture is a hybrid architecture consisting of a pair of GANs: LSTMGAN to simulate the heart rate variability (HRV) pattern of atrial fibrillation (AF) class and DCGAN to generate unique signal morphology. Then ECG signal is produced by changing the length of the generated ECG cycle according to the R-R intervals using cubic spline interpolation.

The hybrid model that Tushar Agarwal. [29] presented also includes two modules, one is focused on generating the real features of heart rate variability (HRV), and the other is focused on generating the real signal morphology. The architecture of both modules is based on WGAN-GP, and also the U-Net architecture is used for their generators which contain convolutional layers. Their proposed framework can also generate signals of arbitrary length by adding a Recurrent Layer (GRU cell) to the U-Net's output.

Tomer Golany. [24], proposed the PGAN (Personalized Generative Adversarial Network) model, which produces special and personalized arrhythmia ECG signals. These generated signals imitate and use the pattern of patients' ECG signals so these generated signals can improve the classification performance of patient-specific ECG signals. Yu-He Zhang. [5], uses a 2D BiLSTM GAN model to generate synthetic ECG signals, which generate a 12-lead ECG signal. This model produces 4 classes LVH, LBBB, ACUTMI, and NORMAL with a high success rate and also shows that using 2D GAN to generate ECG signal is a possible solution to add and enhance real ECG dataset.

FeiZhu. [3] used a GAN-based ECG signal generation method called BiLSTM-CNN GAN. The generator consists of 2 BiLSTM layers and the discriminator has CNN architecture. Finally, they compare their model with 2 different models which are RNN-AE and RNN-VAE and their generated ECGs are morphologically very similar to real signals. Munawara Saiyara Munia. [25] proposed the WGAN (Wasserstein GAN) model to generate ECG signals by training their model with ECG images with fixed dimensions and generating data with similar characteristics.

SHOTA HARADA. [4] proposes a method for generating time series data based on GAN and examines their ability to generate biological signals with their characteristics. This model can generally generate several classes of each time series data from a model and then evaluate the performance of their proposed model. Anne Marie Delaney. [7], introduced GAN architecture which can be used to generate sine waves and synthetic ECG signals. In this research, they focus on the generation of lead II from ECG, and 2 evaluation criteria are used for the suitability of synthetic data for real-world applications and data analysis. They also showed the similarity of generated data to real data and created diversity in synthetic data using the Minibatch layer and evaluated privacy for synthetic ECG signals to ensure that these generated signals preserve patients' privacy.

PU WANG. [8] proposed a framework that consists of two parts: a Data Augmentation model (ACGAN) and a classification model. The first model is a conditional GAN that produces a high-quality ECG signal for the minority class, and then its output is given to the classification model. The second model is used to detect abnormality in cardiac signals. Khondker Fariha Hossain. [9], presented an architecture based on GAN that simultaneously uses ECG signals for producing and also for detecting cardiac abnormalities. Their Generated signals are similar to the real signals and also, they used the loss function related to classification and reconstruction in addition to the GAN loss function

ABDELRAHMAN M. SHAKER. [10], their technique is based on GAN, it balances the dataset and generates lead I from ECG signals that are similar to real signals. This paper has worked on 15 different classes from the MIT-BIH arrhythmia dataset, which is an unbalanced dataset that lowers the accuracy of deep learning models, and also uses two classification approaches: an end-to-end approach and Two-stage Hierarchical approach which are based on deep CNN. Wenqiang Li. [11] proposed a model (SLC-GAN) for the automatic diagnosis of myocardial infarction (MI) that produces single-lead ECG data with high morphological similarity and presented a framework that combines GAN and CNN. HAIXU YANG. [12], presented the ProEGAN-MS model for generating ECG signals with high diversity and accuracy that can generate ECG signals with any resolution without changing the model. Also, the problem of collapse mode is solved in GAN, and the distribution of generated

Subhrajyoti Dasgupta. [13], presented the CardioGAN model, which is based on the GAN network and generates real ECG signals and also has better performance than other methods. Therefore, they proposed an attention-based generator to generate synthetic ECG data which can generate signals with morphological similarity to the real signals by learning more detailed attention-based dependencies and using an efficient loss function based on gradient penalty, which can increase the stability in training. Tomer Golany. [15] introduced a GAN-based model that increases the classification performance by adding generated data to the dataset. Their model learns to generate ECG signals that vary across the ECG signal and other generated ECG signals. Xiaomin Li. [16] has proposed a model based on GAN, both the generator and the discriminator network are built using the transformer encoder, which can completely generate the sequence of time series data with an arbitrary length similar to the real data.

Andrei Furdui. [17] also presented the AC-WGAN-GP model to detect people's emotions by generating ECG and GSR signals. E. Brophy. [18] has proposed a model based on GAN (MVGAN) to generate medical time series and multi-channel ECG signals and also prevented mode collapse with the minibatch layer in the discriminator. The results also show that their model produces signals similar to real signals with diversity, which can guarantee privacy. Shota Harada. [19] has proposed a method for generating time series data that are similar to real data based on the GAN network. Both generator and discriminator are based on recurrent neural networks (LSTM) and also prevent mode collapse by updating weights in the process of training the GAN model.

Jintai Chen. [20], presented a model based on GAN for generating Multi-view ECG signals (ME-GAN). Since heart diseases were concentrated in certain waveforms, they used a mixup normalization to inject disease information exactly in the right place and also used an auxiliary discriminator for monitoring the generator for generating signals with the correct characteristics. They also used the rFID criterion to evaluate the quality of the generated signal and finally, it shows that Their model has performed well in multi-view ECG signal generation. FEI YE. [21], proposed an architecture called RPSeqGAN, which uses gradient penalty and produces signals with different lengths. According to the obtained results, the production data are of high quality and their model is stable. Edmond Adib. [22] also used 2 ways to generate ECG heartbeats, the first model is WGAN-GP, which trains it separately for each class and generates ECG signals for each class separately. The second model is AC-WGAN-GP which gives all available classes to one generator, which is used to generate synthetic heartbeats in all classes. Their Generated signals are similar to the original data and also avoided mode collapse using the Wasserstein function.

Yong-Yeon Jo. [23] proposed the ECGT2T model, which is a deep generative model that combines ten leads from two input leads to simulate a 12-lead ECG. The results also show that the classifiers performed better on the simulated 12-lead ECGs generated with ECGT2T than the models trained on one- or two-lead ECGs in detecting myocardial infarction and arrhythmia. To solve the problem of imbalanced data, Deepankar Nankani.[26] proposed the DCCGAN model to generate ECG signals. Both generator and discriminator networks are based on convolution neural networks, and this model takes the class label information along with its input to generate signals. Also, in the process of training the discriminator, they used a parameter called twist to label 0.1 instead of 0 for generated samples and 0.9 instead of 1 for real samples, which causes faster convergence and solves the problem of vanishing gradient.

Faezeh Nejati Hatamian. [27] investigated the effect of different algorithms for data augmentation and solving imbalanced data, and according to the obtained results, the GAN-based model performs better than the oversampling technique. Their proposed model can Increase the classification accuracy of the atrial fibrillation class by adding generated data.

2. GAN with signals in the transform domain:

In this domain, signals are often transformed using techniques like Fourier transform, wavelet transform or cosine transform, and operations are performed on the transformed signals. Tianjie Lan. [28] presented the model based on the Short-time Fourier transform (STFT) and GAN, which obtained the matrix of coefficients by the Short-time Fourier transform from the ECG signal and used the matrices of different heart rhythm samples to train the GAN model.

Wavelet transform overcomes some of the weaknesses of Fourier transform and ECG signals, which are non-static and dynamic signals. They use this way to analyze the signal in the time-frequency domain, and its advantage over the short-time Fourier transform is that it can solve the problem of the window without changing the STFT by automatically changing the window size. Wavelet transform uses some functions called wavelets, each of which has a different scale. Wavelets occur at a specific moment in time, which can obtain time information in addition to frequency. Then the original signal can be multiplied by the wavelet at different moments, starting with the initial points of the signal and gradually moving the wavelet toward the end of the signal.

Naren Wulana. [6] has presented a model based on waveletGAN(stationary wavelet transform SWT) to generate ECG signals. Among the wavelet transform techniques, they used the stationary wavelet transform (SWT). In this technique, the time decomposition is constant at each level, which means that the time resolution of the time series coefficient corresponds to the original signal part. These obtained coefficients are given as input to GAN and it must perform normalization on them. It feeds each time series coefficient individually to GAN and then trains these GANs to generate time series coefficients. After training the GANs, it uses inverse SWT separately on the generated coefficients to reproduce the ECG parts.

5.2. Autoencoder & Variational Autoencoder Approach

Autoencoder and Variational Autoencoder (VAE) are generative models that were developed before Generative Adversarial Networks (GANs). They serve multiple purposes, including data generation and dimensionality reduction. These models are composed of two main components: an encoder and a decoder. The encoder takes input data, such as digitized ECG shapes, and produces a latent code d, which serves as one of the inputs to the decoder. The purpose of the encoder is to compress the input data into a lower-dimensional representation (the latent code) capturing the most salient features of the input [3].

On the other hand, the decoder takes the latent code d, along with other relevant inputs such as the current output at the current time step and the hidden state at the previous time step. Using this information, the decoder reconstructs the original input data or generates a new sample point waveform. Unlike the encoder, which compresses the input data into a latent code, the decoder works in a sequential manner, using its current output and hidden state, along with the latent code d, to determine the next output. This sequential nature allows the decoder to generate complex and meaningful output sequences, such as time-series data or waveforms [3]. The structure of the Autoencoder is shown in Figure 5.

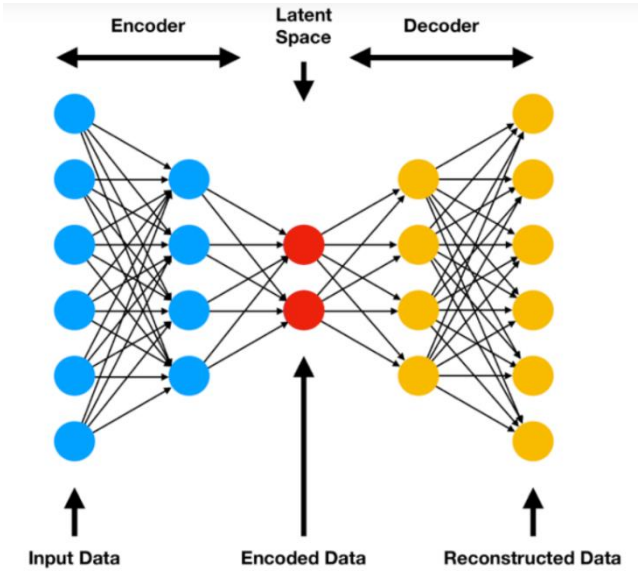

Fig. 5. An Autoencoder Structure

VAE is a type of autoencoder in which the decoder no longer outputs a latent vector, but instead outputs two vectors: a mean vector and a variance vector [3]. The use of the mean and variance vectors in VAE allows for generating new data points by sampling from the latent space. This can allow VAE to have a probabilistic approach to representing each latent attribute for a given input. Unlike traditional autoencoders, VAEs introduce a probabilistic interpretation of the latent space. This means that instead of mapping data to a single point in the latent space, VAEs map data to a probability distribution over the latent space. This probabilistic nature allows for more flexibility and robustness in modeling data uncertainty and also this feature makes VAEs more powerful than the autoencoders.

VV Kuznetsov. [2], presented a method for generating ECG signals and a cardiac cycle using Variational Auto Encoder (VAE) which are similar to the real ECG signals. Moreover,

utilizing this approach, a vector containing 25 novel features is extracted, leading to enhanced diagnostic accuracy for cardiovascular diseases. The generated synthetic signals also prove valuable in addressing the challenge of insufficient labeled ECG signals in supervised learning tasks.

5.3. Wavenet Approach

WaveNet is a deep learning architecture designed specifically for generating audio waveforms. It is a type of autoregressive model, which means it generates audio samples one at a time by conditioning each new sample on the previously generated samples.

Naren Wulana. [6] proposed a model based on WaveNet for generating ECG signals. It is a probability-based autoregression model that was originally used for music and sound generation. The joint probability of the ECG signal can be expressed as a chain of conditional probabilities that each sample Xi in the ECG signal is conditional on the previous samples [6].

This model consists of a series of convolutional layers with residual and skip connections, and it does not include any pooling layer. The final layer utilizes softmax activation for output and classification purposes. The model's notable strength lies in its capacity to generate high-quality long signals, achieved through the utilization of dilated convolution. Additionally, skip connections play a role in accelerating convergence and facilitating the training of deep models. An advantage of this architecture is its ability to generate long ECG signals without compromising the network's performance, even when adding more layers [6]. The details of its architecture are shown in Figure 6.

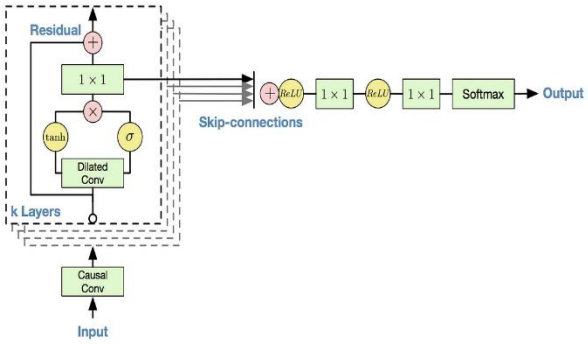

Fig. 6. Wavenet Architecture

This model takes the digitized ECG shape as an input and passes the convolution layers and outputs a sample point waveform [6].

For all the mentioned methods to generate the ECG signal, the table below is summarized:

Reference	Methods	***Dataset***	***Evaluation metrics***
[1]	EmotionalGAN	DECAF	Accuracy
[2]	VAE	LUDB	MMD
[3]	BILSTM-CNN GAN	MIT-BIH arrhythmia	PRD, FD, RMSE
[4]	Recurrent GAN	ECG200, Twolead ECG	DTW
[5]	2D BILSTM GAN	PTB-XL, CCDD, CSE, Chapman	PRD, RMSE
[6]	SpectroGAN , WaveNet,WaveletGAN	MIT-BIH arrhythmia	GAN train, GAN test
[7]	4CNN GAN , 4CNN BILSTM GAN	MIT-BIH arrhythmia	MMD , DTW
[8]	ACGAN	MIT-BIH arrhythmia, CPSC	Euclidean distance, Kullback Leibler, Pearson correlation
[9]	ECG-Adv-GAN [9]	MIT-BIH arrhythmia	Pearson correlation, structural similarity, RMSE, MSE
[10]	GAN [10]	MIT-BIH arrhythmia	Specificity, sensitivity
[11]	SLC-GAN	PTB Diagnostic ECG	KID, FID
[12]	ProEGAN-MS	MIT BIH Arrhythmia	Euclidean distance, Kullback Leibler, Pearson correlation, DTW, GAN train, GAN test
[13]	CardioGAN	MIT-BIH Arrythmia , MIT-BIH normal sinus rhythm	PRD, RMSE, FD, DTW
[14]	LSTMGAN+ DCGAN	physioNet challenge 2017, physioNet challenge 202	sensitivity, specificity
[15]	DCGAN	MIT BIH Arrhythmia	AUC

Reference	Methods	***Dataset***	***Evaluation metrics***
[16]	TTS-GAN	PTB Diagnostic ECG	PCA,t-SNE, cosine similarity, Jensen-Shannon distance
[17]	AC-WGAN-GP	CASE	weighted F1
[18]	MV-GAN	MIT BIH Arrhythmia	MMD, DTW
[19]	LSTMGAN	ECG200, EEG	Accuracy
[20]	ME-GAN	PTB Diagnostic ECG, Tianchi ECG	rFID,precision-recall curve
[21]	RP-Seq-GAN	MIT-BIH Arrhythmia	MMD, Euclidean distance, DTW, Relative Entropy, Time Wrap Edit Distance, soft DTW
[22]	AC-WGAN-GP, WGAN-GP	MIT-BIH Arrhythmia	DTW
[23]	ECGT2T	PTB-XL, CUSPH	missing errors for domain and R peaks
[24]	PGAN	MIT-BIH Arrhythmia	AUC
[25]	WGAN	PTB Diagnostic ECG, MIT-BIH normal sinus rhythm	FID, F-measure
[26]	DCCGAN	MIT-BIH Arrhythmia	FID, MMD, DTW, Euclidean distance, KLD, TWED
[27]	DCGAN	PhysioNet/CinC Challenge 2017	Accuracy, F1-score
[28]	CNNGAN	MIT-BIH Arrhythmia	sensitivity, specificity, F1 score
[29]	Cardiac Gen	WESAD	RMSE, RMSSD

6. ECG'S DATASETS

A good dataset for generating electrocardiogram (ECG) signals is the foundation for building accurate, robust, and ethical machine learning models that can generalize well and provide valuable insights and predictions in various domains. It should include the ECG signal data with time information and lead details, along with labeled annotations for different ECG types. It should have metadata, noise, and artifacts, representing both normal and abnormal cases while protecting patient privacy. The dataset also should be of sufficient size, properly split for training, validation, and testing, and provided in a standard format.
Several studies have been conducted on a dataset that researchers collected from electrocardiogram (ECG) signals available in hospitals. However, for better comparability with others, it is preferable to use common datasets.
The common datasets that are used to generate ECG signals and evaluate these generated signals are as follows:

1) *MIT-BIH arrhythmia: This database contains 48 half-hour records with a sampling rate of 360 Hz. Its records include two channels which are, MLII and v5. It also includes 15 different classes for classifying ECG signals [30].*

2) *LUDB: Lobachevsky University Electrocardiography Database (LUDB) is an ECG signal database that consists of 200 10-second 12-lead ECG signal records with a sampling rate of 500 Hz [31].*

3) *ECG200: It consists of 200 samples of an ECG series. Each series traces the electrical activity recorded during one heartbeat. it also contains two classes that are a normal heartbeat and a Myocardial Infarction [32].*

4) *PTB Diagnostic ECG: This database contains 549 records from 290 subjects. Each subject is represented by one to five records. Each record includes 15 channels: the 12 leads ECG signal (i, ii, iii, avr, avl, avf, v1, v2, v3, v4, v5, v6) and 3 Frank lead ECGs (vx, vy, vz) with a sampling rate of 1000 Hz. It also contains 9 diagnostic classes for ECG signals [33].*

5) *CPSC: The China Physiological Signal Challenge (CPSC) 2018 contains 9,831 records from 9458 patients with a length of 7-60 minutes. Also, it includes nine ECG categories, one of them for normal ECG type and the other for abnormal types [34].*

6) *MIT-BIH normal sinus rhythm: This database includes 18 long-term ECG records which are normal and there is no sign of arrhythmia in them [35].*

7) *PTB-XL: This dataset contains 21837 clinical 12-lead ECG records from 18885 patients. These records have 10 seconds length with a sampling frequency of 400 Hz. It also includes 5 main categories and 24 subclasses for class labels [36].*

8) Chapman: This dataset contains 10 seconds 12-lead ECGs of 10,646 patients with a 500 Hz sampling rate that features 11 common rhythms and 67 additional cardiovascular conditions [37].

7. Evaluation Metrics

Some evaluation criteria have been used to evaluate the generated ECG signals and the correctness of these signals in terms of similarity to real signals. Because it is important to evaluate these signals for adding them to the data set and use them as an augmented dataset to classify the signals.
In general, we have two categories of criteria for evaluating ECG signals:

- Criteria for evaluating the accuracy and performance of the ECG classification:

1) Precision:

Precision is a performance metric that measures the accuracy of positive predictions made by a model. It is calculated by dividing the number of true positive predictions (correctly predicted positive instances) by the sum of true positive predictions and false positive predictions (instances predicted as positive but actually negative).

$$\text{Precision} = \frac{TP}{TP + FP}$$

The precision value ranges between 0 and 1 (or 0% and 100%). The maximum value of 1 (or 100%) is achieved when there are no false positive predictions, indicating that all positive predictions made by the model are correct. The minimum value of 0 is obtained when there are no true positive predictions, meaning that the model did not correctly identify any positive instances.

2) Recall:

Recall, also known as True Positive Rate (TPR) or Sensitivity, is a performance metric that measures the ability of a model to correctly identify positive instances out of all instances that should have been predicted as positive. It is calculated by dividing the number of true positive predictions (correctly predicted positive instances) by the sum of true positive predictions and false negative predictions (instances that were positive but wrongly predicted as negative).

$$\text{Recall} = \text{Sensitivity} = \frac{TP}{TP + FN}$$

The recall value also ranges between 0 and 1 (or 0% and 100%). The maximum value of 1 (or 100%) is achieved when the model correctly identifies all positive instances, and there are no false negatives, meaning that it doesn't miss any positive cases. The minimum value of 0 is obtained when the model fails to identify any positive instances correctly.

3) Accuracy:

Accuracy is a performance metric used in classification tasks to determine the proportion of correct predictions made by a model out of the total number of predictions. It is calculated by dividing the number of true positive predictions (correctly predicted positive instances) and true negative predictions (correctly predicted negative instances) by the total number of predictions (both true positives and true negatives combined).

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

In other words, accuracy measures how well a model classifies both positive and negative instances correctly. Accuracy values range between 0 and 1 (or 0% and 100%), where a value of 1 (or 100%) indicates a perfect classification model that predicts all instances correctly, and a value of 0 indicates a model that fails to make any correct predictions.

4) F1 Score:

When you want your evaluation criteria to be an average of Recall or Precision criteria, you can use the harmonic average of these two criteria, which is called the f1-score criteria.

$$\text{F1 Score} = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

5) Specificity:

Specificity, also known as True Negative Rate (TNR), is a performance metric used in classification tasks to measure the ability of a model to correctly identify negative instances out of all instances that should have been predicted as negative. It is calculated by dividing the number of true negative predictions (correctly predicted negative instances) by the sum of true negative predictions and false positive predictions (instances that were negative but wrongly predicted as positive).

$$\text{Specificity} = \frac{TN}{TN + FP}$$

Similar to precision and recall, specificity (TNR) also ranges between 0 and 1 (or 0% and 100%). The maximum value of 1 (or 100%) is achieved when the model correctly identifies all negative instances, and there are no false positives, meaning that it doesn't misclassify any negative cases. The minimum value of 0 is obtained when the model fails to identify any negative instances correctly.

- Criteria for evaluating the generated ECG signal in terms of similarity to the real ECG signal:

1) DTW:

The Dynamic Time Warp (DTW) similarity measure is a well-established approach for quantifying the similarity or dissimilarity between time series curves. By finding the optimal alignment of two given time series with minimum cost, DTW calculates their degree of similarity [13].
It is a classical and widely used method to estimate the dissimilarity between two time series and has shown consistent superiority over other similar benchmarks. DTW achieves its alignment by warping the series along the time axis, optimally aligning corresponding points, and subsequently computing the distance between the two-time series. This technique allows for effective comparison and evaluation of the similarity between time series data [7].

$$D_{i,j} = f(x_i, y_j) + \min\{ D_{i,j-1}, D_{i-1,j}, D_{i-1,j-1}\}$$

2) MMD:

Maximum Mean Discrepancy (MMD) is a statistical measure used to assess the dissimilarity between two probability distributions, denoted as Pr and Pg. MMD is commonly employed when there are sample datasets available from both distributions, which are drawn independently. By comparing the means of features from these two datasets, MMD quantifies the discrepancy between the distributions [7].

3) PRD:

To quantify the distortion between real and generated signals, the root mean square difference percentage (PRD) is a widely used method. PRD is a common distortion measurement technique that provides a quantitative assessment of the dissimilarity between the two sets of signals. The PRD calculates the root mean square difference between corresponding data points in the real and generated signals, expressed as a percentage of the signal amplitude. It effectively measures the average percentage difference between the values of the real and generated signals, providing valuable insights into the fidelity of the generated data compared to the original real data [13].

$$\mathrm{PRD} = \sqrt{\frac{\sum_{t=1}^{T}(x_t - x_t')^2}{\sum_{t=1}^{T}(x_t)^2}} \times 100$$

4) FD:

The Frechet distance (FD) is a similarity measure between curves that takes into account the spatial arrangement and the order of points along the curves. It is particularly useful for comparing time series data or any other type of curve that have a natural ordering of points. The FD calculates the minimum traversal distance between two curves, considering all possible parameterizations of the curves. In essence, it measures how far apart two curves are while allowing them to be traversed at varying speeds and directions. In the context of evaluating the performance of a generative model, a lower Frechet distance typically indicates higher quality and diversity of the generated results. A smaller FD value means that the generated curves closely match the original data in terms of their shape and order of points, reflecting better accuracy and fidelity in the generated samples. [3]

5) Euclidean distance (ED):

With this metric, the distance between two probability distributions of real and generated signals can be obtained [21].

$$d_{ED}(P_g, P_r) = \sqrt{\sum_{k=1}^{N}(x_k - y_k)^2}$$

A lower value for it indicates a high quality of the generated signal.

6) KLD:

It is also called a relative entropy that measures the difference between two probability distributions [26].

$$D_{KL}(P_r \parallel P_g) = \sum_{i=1}^{N} P_r(x_i) \cdot \log \frac{P_r(x_i)}{P_g(x_i)}$$

7) Pearson correlation:

It can be used to obtain a linear correlation between two signals. Its value is between 1 and -1, and the higher this value is, the more correlation there is between two signals, so the quality of the generated signal is higher [26].

$$PC(X,Y) = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}}$$

8) Cosine similarity:

It measures the similarity between two vectors, a real signal feature vector, and also a generated signal feature vector [16].

$$\cos\_sim_{ab} = \frac{f_a \cdot f_b}{|f_a||f_b|} = \frac{\sum_{i=1}^{m} f_{ai} f_{bi}}{\sqrt{\sum_{i=1}^{m} f_{ai}^2}\sqrt{\sum_{i=1}^{m} f_{bi}^2}}$$

9) Jensen-Shannon distance:

It measures the similarity between two probability distributions. It is based on the Kullback Leibler divergence, with some differences, that it is symmetric and also it always has a finite value.

It calculates the distance between real and generated signal features of the same class [16].

$$jen\_sim_i = \sqrt{\frac{D(f_{i\_real} \parallel m) + D(f_{i\_syn} \parallel m)}{2}}$$

10) MSE:

Mean Squared Error represents the average of the squared difference between the real and predicted values in the data set.

11) RMSE:

Root Mean Squared Error is the square root of Mean Squared error.

8. Summary

According to the methods presented for generating ECG signals, it can be understood that by balancing the corresponding dataset, the classification performance in ECG signals can be improved. Various methods for generating ECG signals have been mentioned, which utilize mathematical and deep learning approaches. These methods have been categorized based on their generative approaches, and the architectures used in each study have been provided. As mentioned before, a major challenge in ECG signal generation is the morphological similarity between these synthetic signals and real signals, and the generated data distribution should be similar to real signals. Furthermore, in some methods, it has been observed that creating diversity alongside morphological similarity is crucial, and by generating these signals and

applying techniques such as adding noise, the privacy of individuals can be preserved. Additionally, Common datasets are utilized for ECG signal generation, and specific evaluation metrics are introduced to assess the quality of the generated signals. By evaluating the generated ECG signals, higher-quality subsets can be added to the dataset, ultimately enhancing its overall performance. The incorporation of these generated ECG signals into the dataset and balancing it contributes to detecting cardiac diseases and improves classifier performance, as indicated by the results obtained from the articles. In conclusion, synthetic ECG signals generated through appropriate methodologies and evaluation play a crucial role in more accurate diagnosis of cardiovascular diseases, enhancing classification performance, and ensuring the privacy and integrity of patient data.

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