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Leiserson Saxe

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FPGA

Combinatorial rippling

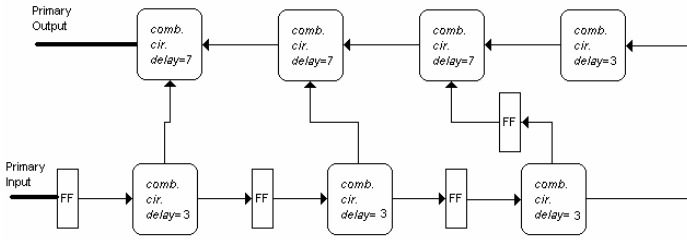
Leiserson Saxe [ , ]

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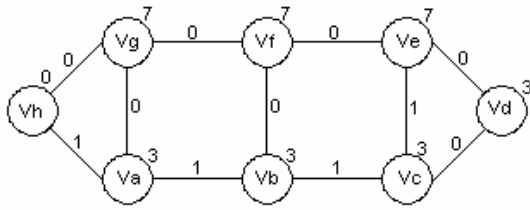
$W'_{ij}$

$$W'_{ij} = W_{ij} + r(j) - r(i)$$

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```

For i=1 to No_of_vertices
begin
  find weights
  look at comb. parts
  find delays
  find violating vertices(delay > ϕ)
  if ( none ) then
    return successful
  else
    retime violating vertices
end

```

B A B A

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$W_{ij}$   $r(i)$   $d(i)$   $W_{ij}$

$d(i)$   $j$   $i$

$i$   $r(i)$   $i$

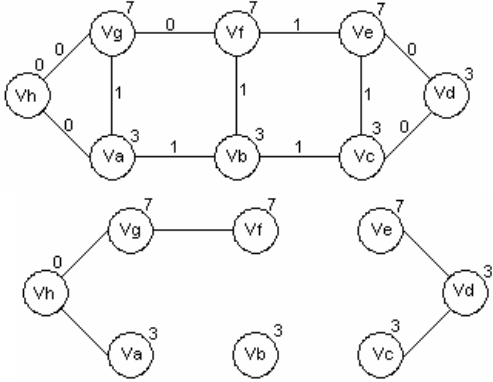
B A

$0 < W_{ij}$

$r(i)$

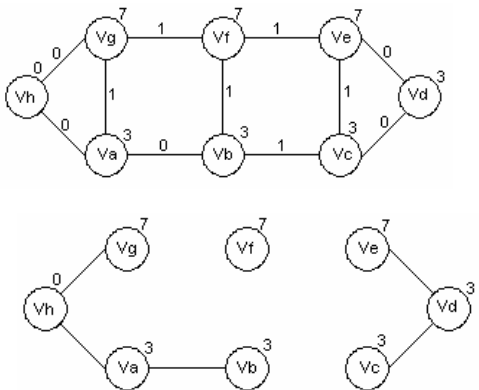
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$$\begin{aligned}
 W_{ha} &= 1+0-1=0 & W_{ce} &= 1+0-0=1 \\
 W_{ab} &= 1+0-0=1 & W_{de} &= 0+0-0=0 \\
 W_{ag} &= 0+1-0=1 & W_{ef} &= 0+1-0=1 \\
 W_{bc} &= 1+0-0=1 & W_{fg} &= 0+1-1=0 \\
 W_{bf} &= 0+1-0=1 & W_{gh} &= 0+1-1=0 \\
 W_{cd} &= 0+0-0=0
 \end{aligned}$$

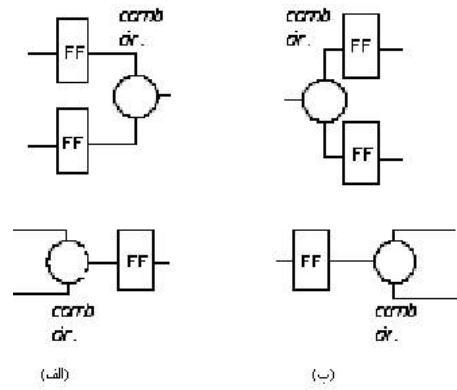


a h g

$$\begin{aligned}
 W_{ha} &= 0+1-1=0 & W_{ce} &= 0+0-0=0 \\
 W_{ab} &= 1+0-1=0 & W_{de} &= 0+0-0=0 \\
 W_{ag} &= 1+1-1=1 & W_{ef} &= 1+0-0=1 \\
 W_{bc} &= 1+0-0=1 & W_{fg} &= 0+1-0=1 \\
 W_{bf} &= 1+0-0=1 & W_{gh} &= 0+1-1=0 \\
 W_{cd} &= 1+0-0=1
 \end{aligned}$$



g,h,a,b

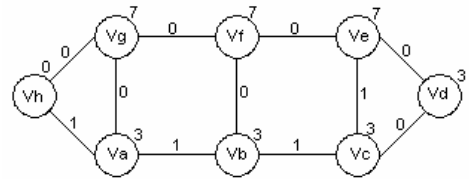


Gate Join ( )-

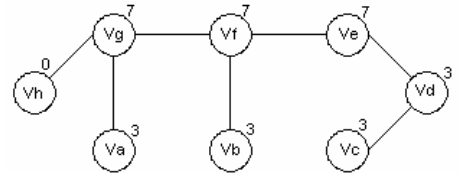
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Fork

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Vh, Vg, Vf, Ve, Vd, Vc

++++ =

(r(i)=1)

h g f

$$W'_{ij} = W_{ij} + r(j) - r(i)$$



**Algorithm Sam-1 :**

// Improvement of Leiserson and Saxe retiming algorithm:

```

Use the standard retiming algorithm for the problem
If ( solution was not found ) then
  while (solution is not found )
    add one register level to the input edge of the
    graph
    use the standard retiming algorithm for the
    new
    graph
  end while
else
  while ( it's possible )
    delete one register level
    use retiming algorithm for the new graph
  end while
  add one register level to the input edge of the
  graph
  use the standard algorithm for the new graph
end if

```

else

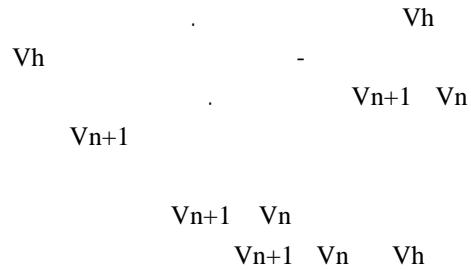
B A B A

n n

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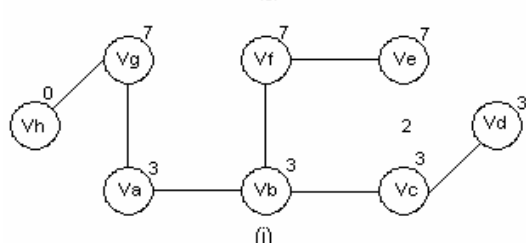
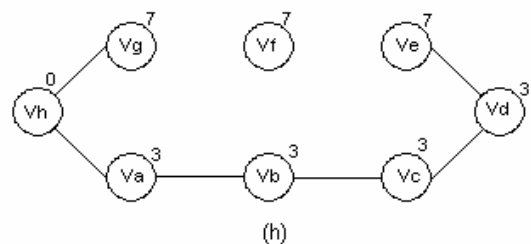
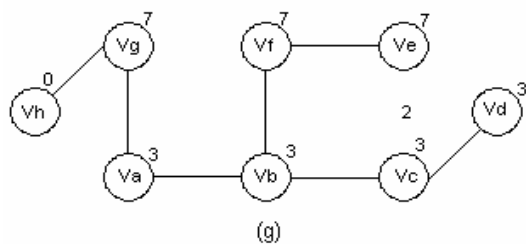
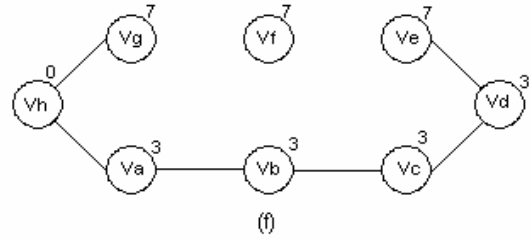
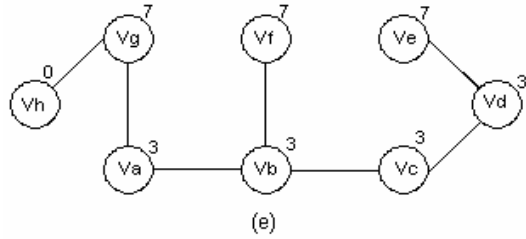
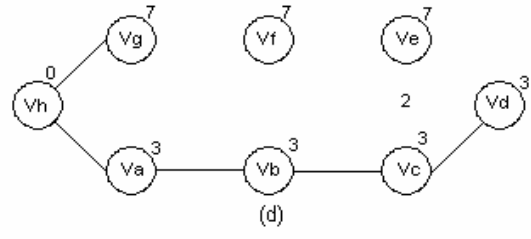
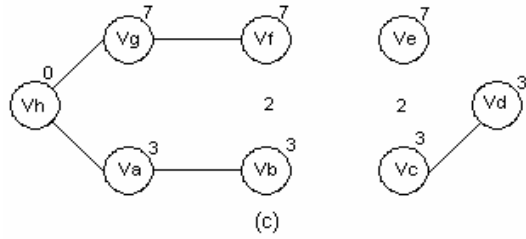
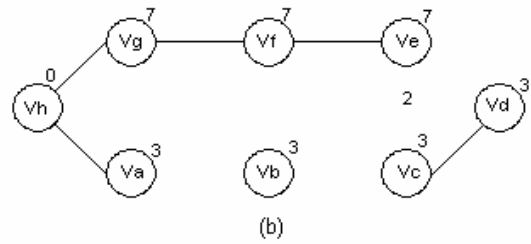
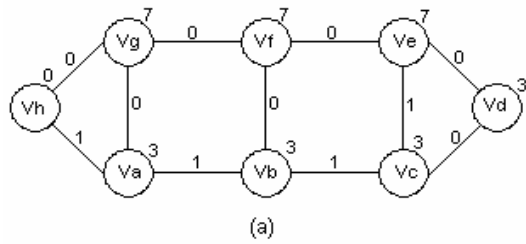
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W<sub>ha</sub>  
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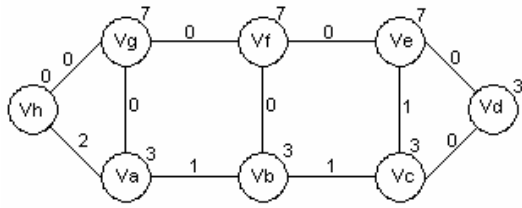
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$$\sum_{i=1}^k m_i = n \quad m_1, m_2, \dots, m_k$$

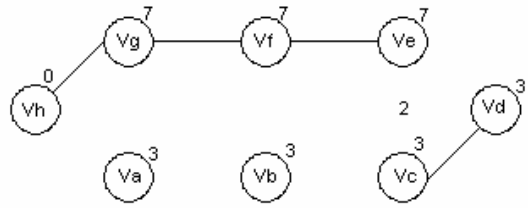
$$\forall 1 \leq i \leq k : m_i \leq n \quad [ ] \quad n$$

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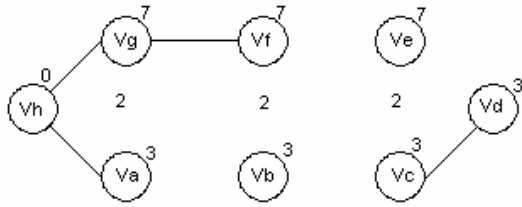
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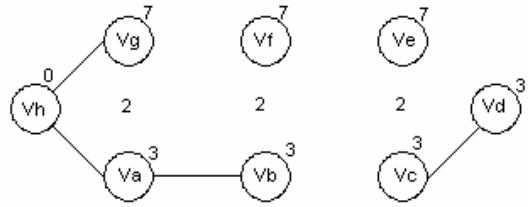
(a)



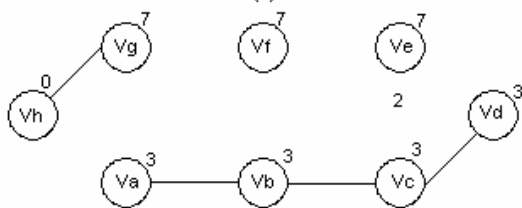
(b)



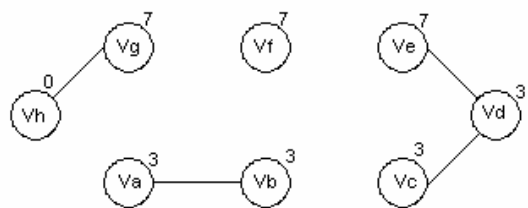
(c)



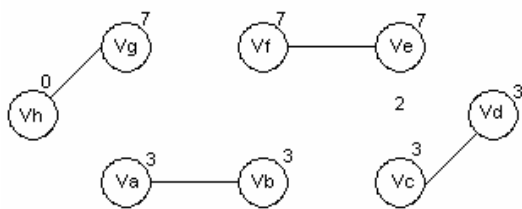
(d)



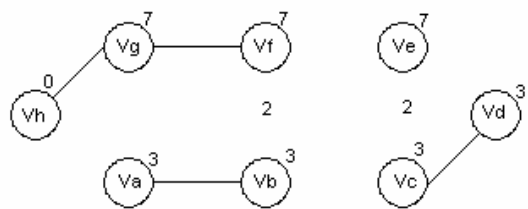
(e)



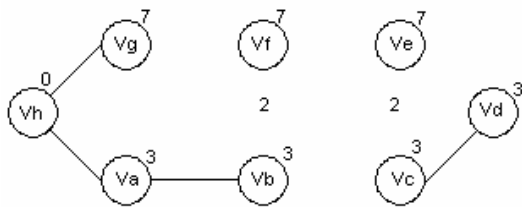
(f)



(g)



(h)



(i)

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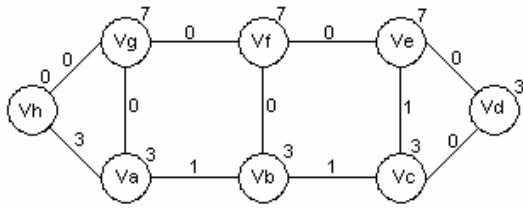
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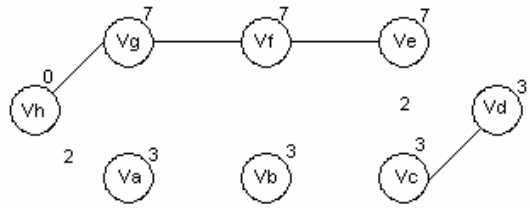
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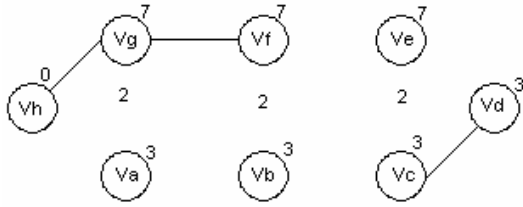
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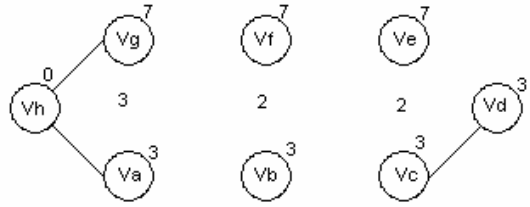
(a)



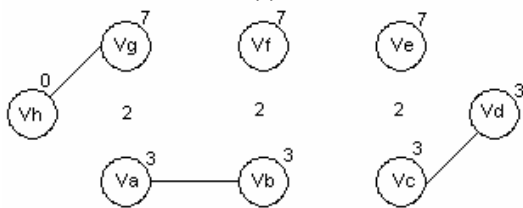
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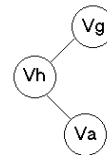
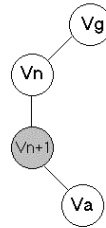
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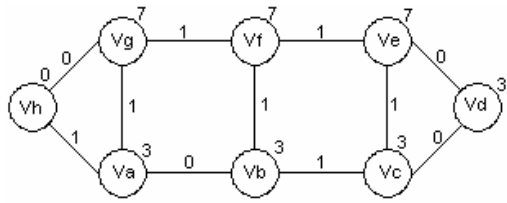
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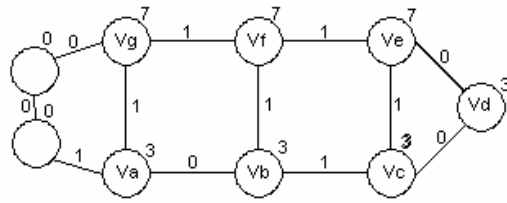
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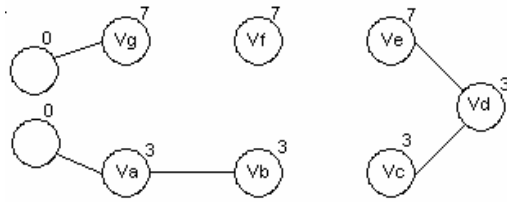
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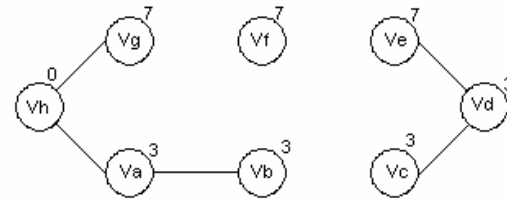
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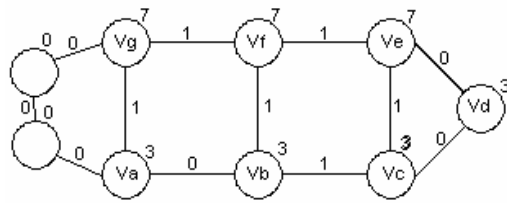
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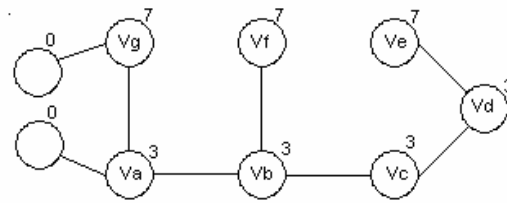
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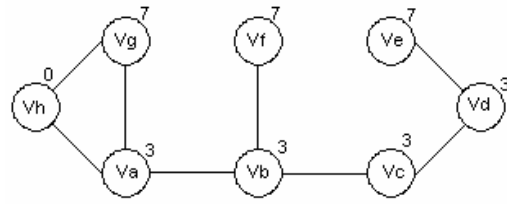
(d)



(e)



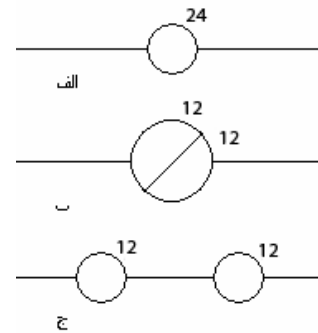
(f)



(g)

(g)

x



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$$mn > (n+1)(m-a)$$

$$(n+1)(m-a)$$

$$m < (n+1)a$$

$$m-na-a < 0$$

$$\frac{m}{n+1} < a$$

x

x

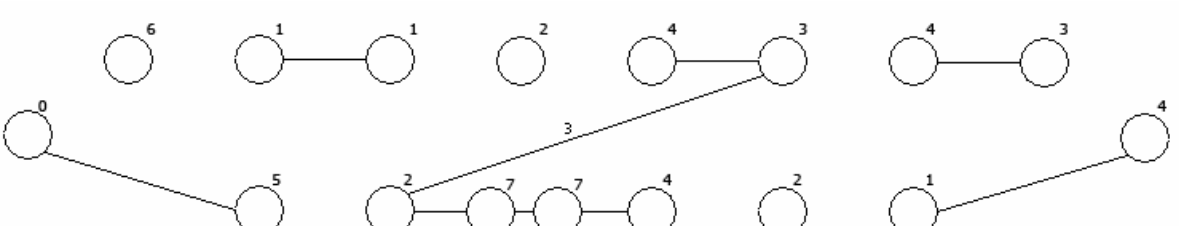
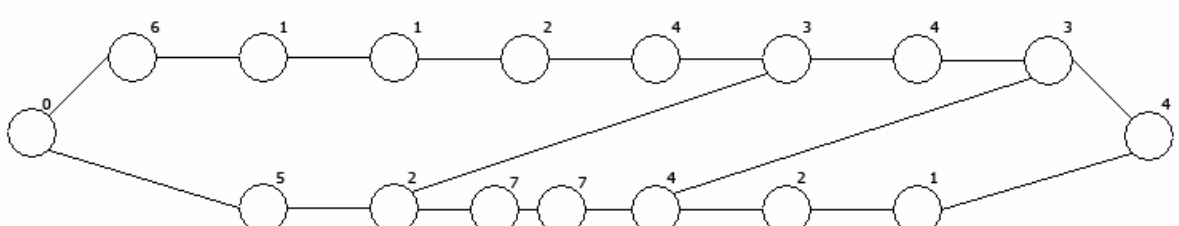
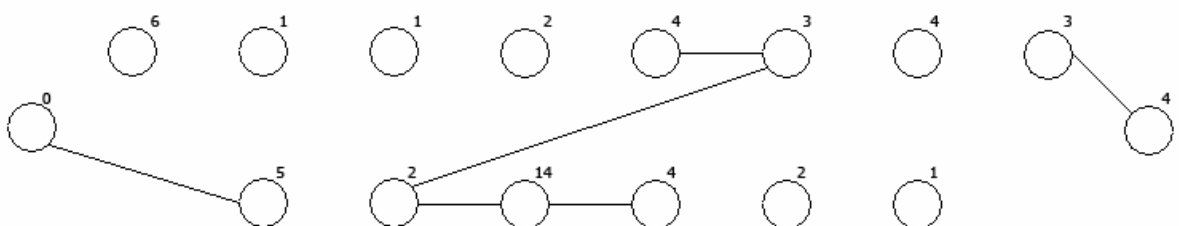
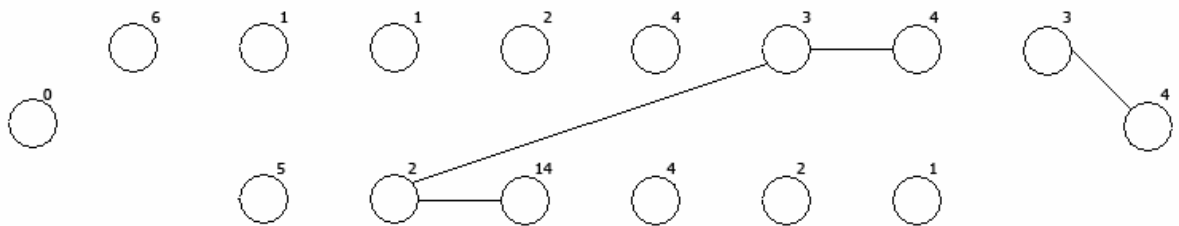
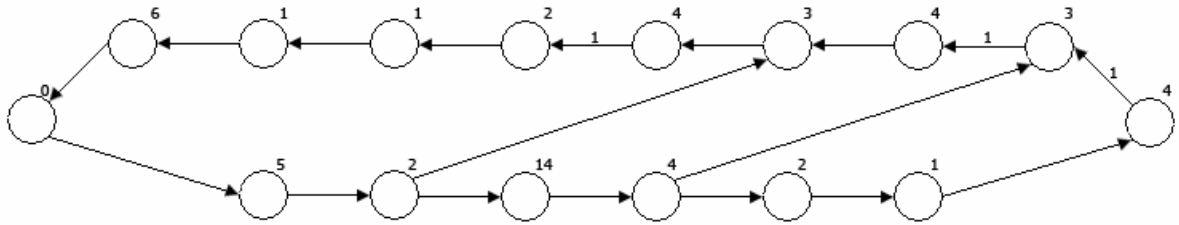
x

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$$\frac{m}{(n+1)}$$

n m

$$\frac{m}{(n+1)}$$



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<sup>1</sup> Retiming

<sup>2</sup> Trade off

<sup>3</sup> Critical path

lus

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Silicon Valley

CAD

Msedighi@ce.aut.ac.ir

VLSI



Testable design CAD

[SFarrokhi@specialsolutions.ca](mailto:SFarrokhi@specialsolutions.ca)

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